Keep it Simple Solution for Nrich

Myself **Shubhangee (Facilitator)**had worked collaboratively on '**Keep it Simple**' with a group of 10 students of 4^{th} and 5th grade, in online mode, in '**Ganit Kreeda', Vicharvatika, India.** The names of the students are:

Nikhil, Karthik, Vishnuvardhan, Sehar, Dhruv, Aarya, Kanav, Atharvan, Keeya, Aaradhya, Rishaan.

Charlie thought he'd spotted a rule and made up some more examples.
1/2=1/10+1/20
1/3=1/4+1/12
1/3=1/7+1/21
1/4=1/5+1/20

Kanav explained it as:

Ans1.Charlie's rule is that for example if we take a fraction 1/n, he just takes one more unit fraction for example 1/2 and then multiplies it with 1/n which gives him 1/2n. But that's not working always.

Ans 2. The outcomes made by Charlie in the red text above are
not correct .

Ans3.The observation is that for any unit fraction, the denominators of other 2 fractions which we are adding are greater than the denominator for which we are solving.

Ans 4. Here are some other correct examples

⅙=1/9+1/18

⅙=1/24+⅛

⅛=1/24+1/12

Ans.5 For the correct outcomes the following formula is true-

1/n=1/n+1+1/n^2+n

Here is the complete calculation of charlie's answers
1/8=1/9+1/72
1/8=1/10+1/40
1/8=1/11+3/88
Can all unit fractions be made in more than one way like this?
Yes.

The first way of expressing ⅙ has used this way-1/n=1/n+1+1/n^2+n

The second way of expressing ⅙ has used the way of equivalent
fractions, for example 3/18 is the equivalent fraction of ⅙
It is quite simple that -
1/6 = 3/18 =1/18+2/18 = 1/18+1/9

To find other ways, the equivalent fraction rule is again used such that 1/6 = 4/24
It's quite simple that 1/6 = 4/24 = 1/24 + 3/24 = 1/24 + ⅛

How to find all possible ways to write 1/6 using equivalent Fraction is explained here:

we found out that –

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$ $\left\{ \frac{1}{6} = \frac{7}{42} = \frac{6+1}{42} = \frac{6}{42} + \frac{1}{42} = \frac{1}{7} + \frac{1}{42} \right.$

$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$ $\left\{ \frac{1}{6} = \frac{8}{8 \times 6} = \frac{6+2}{8 \times 6} = \frac{1}{8} + \frac{1}{24} \right\}$

$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$ $\left\{ \frac{9}{9 \times 6} = \frac{6+3}{9 \times 6} = \frac{1}{9} + \frac{1}{18} \right\}$

$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$ $\left\{ \frac{10}{10 \times 6} = \frac{6+4}{10 \times 6} = \frac{1}{10} + \frac{1}{15} \right\}$

$\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$ $\left\{ \frac{12}{12 \times 6} = \frac{6+6}{12 \times 6} = \frac{1}{12} + \frac{1}{12} \right\}$

We analysed all these examples, in the reverse way as shown in brackets. We then realised that we need to check all possible numerators where numerator = 6+1, 6 + 2, 6+3,…6+6. Or 6+m where m can be anything from 1 to 6.

It will work only for those sums(or numerators), where the addend (m) is the factor of (6+m)6 = 36 + 6m.

We then generalised it for any unit fraction 1/n.

According to above observation, it will work only for those sums(or numerators), where m(addend) is the factor of (n+m)n or m is a factor of n^2 + nm.

This is true only if m divides n^2 (as m divides nm.)

Also we know that m can take values from 1 to n. That means all factors of n^2 which are smaller than or equal to n, can be chosen as m.

This theory was tested for 1/12.

First we anticipated all possibilities :

Now we have to find out m such that m divides n(n+m) or 12(12+m)= 144 + 12m…{as here n=12}

All factors of 144 smaller than or equal to 12 are 1,2,3,4,6,8,9,12.

So, there are 8 ways to write 1/12. As we want distinct unit fractions, last one is not correct. So, there are 7 ways to write 1/12 as sum of 2 distinct fractions.

This is shown here:

12+1, 12+2, 12+3, 12+4, 12+6, 12+8, 12+9, 12+12.

Factors of 144 which are smaller than or equal to 12 are 1, 2, 3, 4, 6, 8, 9 & 12.

⇒ there are 8 ways to write 1/12

$$\frac{1}{12} = \frac{12+1}{12\times13} = \frac{12}{12\times13} + \frac{1}{12\times13} = \frac{1}{13} + \frac{1}{12\times13} = \frac{1}{13} + \frac{1}{156}$$

$$\frac{1}{12} = \frac{12+2}{12\times14} = \frac{12}{12\times14} + \frac{2}{12\times14} = \frac{1}{14} + \frac{1}{84}$$

$$= \frac{12+3}{12\times15} = \frac{12}{12\times15} + \frac{3}{12\times15} = \frac{1}{15} + \frac{1}{60}$$

$$= \frac{12+4}{12\times16} = \frac{12}{12\times16} + \frac{4}{12\times16} = \frac{1}{16} + \frac{1}{48}$$

$$= \frac{12+6}{12\times18} = \frac{1}{18} + \frac{1}{36}$$

$$= \frac{12+8}{12\times20} = \frac{1}{20} + \frac{1}{30}$$

$$= \frac{12+9}{12\times21} = \frac{1}{21} + \frac{1}{28}$$

$$= \frac{12+12}{12\times24} = \frac{1}{24} + \frac{1}{24}.$$

Keep it Simple Solution for Nrich

Myself **Shubhangee (Facilitator)**had worked collaboratively on '**Keep it Simple**' with a group of 10 students of 4^{th} and 5th grade, in online mode, in '**Ganit Kreeda', Vicharvatika, India.** The names of the students are:

Tejas, Valerie, Varun, Kaumudi, Dhwani, Ira, Prathamesh, Adhrit, Delissaa, Riyansh.

1. Can you describe Charlie's rule?

Ans: Dhwani explained it as:

Charlie's rule was to take a unit fraction that was immediately smaller than the fraction given and subtract. Then he added the difference and the first unit fraction together where sum was given unit fraction

2. Are all his examples correct?

Ans: No, the first and the third are not correct.

3. What do you notice about the sums that are correct?

Ans: The given unit fraction's denominator and the first unit fraction's denominator are consecutive numbers.

Dhwani said that…In the sums that are correct, the first unit fraction was immediately lesser than the answer of the sum.

4. Find some other correct examples. How would you explain to Charlie how to generate lots of correct examples?
   1/5 = 1/6 + 1/30
   1/7 = 1/8 + 1/56
   1/12 = 1/13 + 1/ 156

Ans: Take two consecutive numbers and multiply them together. Write the consecutive numbers and the product as unit fractions. E.g. 3x 4= 12 & 1/3 = ¼ + 1/12. Or 8x9 = 72 and 1/8 = 1/9 + 1/72.

Adhrit spotted the pattern from Charlie's examples as 1/n = 1/(n+1) + 1/n(n+1).

Dhwani said that for any given unit fraction like 1/8, take the biggest possible unit fraction smaller than 1/8…which is 1/9. And then subtract 1/9 from 1/8 to get 1/72.

In general,

For given any unit fraction 1/n, Charlie has selected the biggest unit fraction

1/(n+1) smaller than the given fraction. And then subtract 1/{n+1} from 1/n to

get another unit fraction 1/n(n+1).

Adhrit explained why this will always give unit fraction.

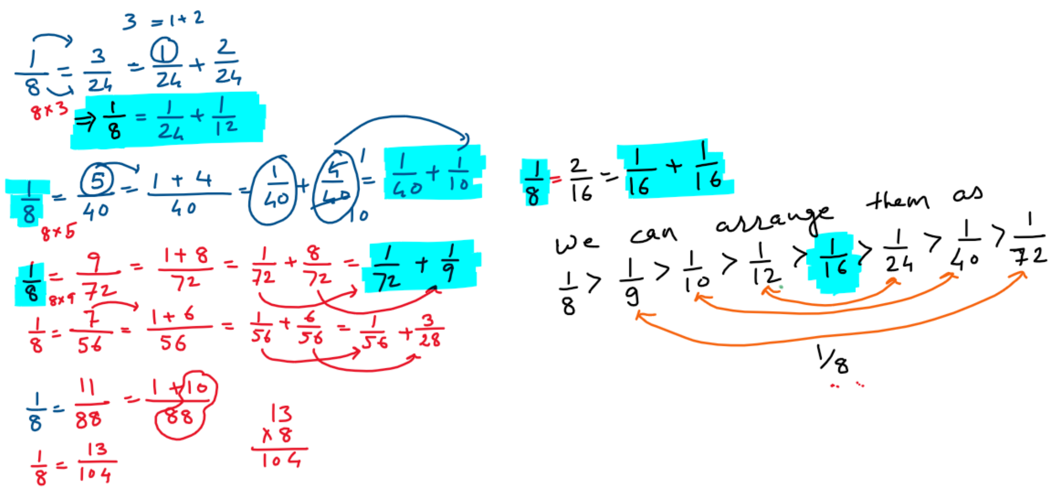
As n and n+1 are consecutive numbers and as 2 consecutive numbers are always co-prime, LCM of n and (n+1) is always 1.

1/n – 1/(n+1) = [(n+1) – n]/n(n+1) = 1/n(n+1)

6. Charlie tried to do the same with ⅛. Can you finish Charlie's calculations to see which ones work?

Children used trial and error for getting all possible ways starting from 1/9 till 1/16 which is half of 1/8. They used concept of Equivalent fraction to find all possible solutions.

Adhrit chose an odd number to multiply the denominator(i.e. 8 here) to get even denominator. For ex: 1/8 = 1/8x3 = 1/8x3 + 2/8x3= 1/24 + 1/12. He tried it with all odd multipliers to get even denominator. It is shown here.



1/8=1/9+1/72

1/8=1/10+1/40

1/8=1/11+… (Does not work)

1/8= 1/12 + 1/24

1/8=1/13+… (Does not work)

1/8=1/14+… (Does not work)

1/8=1/15+… (Does not work)

1/8=1/16+1/16 (Does not work as these are not distinct.)

8. Choose different unit fractions of your own to test out your theories.

1. 1/9 = 1/10 + 1/90

2. 1/9 = 1/11 + 2/99 (Does not work)

3. 1/9 = 1/12 + 1/36

4. 1/9 = 1/13 + 1/107 (Does not work)

1/9 = 1/14 + 1/126 (Does not work)

……………………………………

1/9 = 1/18 + 1/18

Children observed that:

1. Can you describe Charlie's rule? - One of the addends' denominators must be a product of the other addend's denominator and the sum's denominator.

2. Are all his examples correct? - No, the second one and the fourth one are not correct.

3. What do you notice about the sums that are correct? - The addends' denominator and the given sum's denominator are consecutive numbers.

4.Find some other correct examples..

$$\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$$

$$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

5. How would you explain to Charlie how to generate lots of correct examples? - Take two consecutive numbers and multiply them together. Ex. 7 x 8 = 56.

Write the consecutive numbers and the product as unit fractions.

Write the sum of the two smaller fractions as the sum of the biggest fraction.

I.e 1/7 = ⅛ + 1/56

Most of the children generalised the above as:

Step 1: $\frac{1}{a}$ write a unit fraction

Step 2: $\frac{1}{a} = \frac{1}{a+1} + x$ write the successor.

Step 3. $\frac{1}{a} = \frac{1}{a+1} + \frac{1}{a(a+1)}$

6. Charlie tried to do the same with ⅛. Can you finish Charlie's calculations to see which ones work?

1/8=1/9+1/72

1/8=1/10+1/40

1/8=1/11+3/88 (Does not work)

7. Can all unit fractions be made in more than one way like this? - Yes.

8. Choose different unit fractions of your own to test out your theories.

1. 1/9 = 1/10 + 1/90
2. 1/9 = 1/11 + 2/99 (Does not work)
3. 1/9 = 1/12 + 1/36

Arya observed that Charlie's technique worked only if the difference between the denominators of the first addend and the sum divides their product.

1/9 = 1/10 + 1/90 works because (10 - 9) divides 10*9. The quotient (10*9 / 1 = 90) is the denominator of the other addend.

1/9 = 1/12 + 1/36 works because (12 - 9) divides 12*9. The quotient (12*9 / 3 = 36) is the denominator of the other addend.

1/9 = 1/11 + 2/99 does not work (11-9) does not divide 11*9

9. **Charlie tried to do the same with 1/8. Can you finish Charlie's calculations to see which ones work?**

1. 1/8=1/9+1/72
2. 1/8=1/10+1/40
3. 1/8=1/11+3/88 (Does not work)

1/8=1/9+1/72 works because (9-8) divides 9*8. The quotient (9*8 / 1 = 72) is the denominator of the other addend.

1/8=1/10+1/40 works because (10-8) divides 10*8. The quotient (10*8 / 2 = 40) is the denominator of the other addend.

1/8=1/11+3/88 does not work because (11-8) does not divide 11*8.

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USING EQUIVALENT FRACTIONS

Reyansh and Anirved came up with this method. They figured out that if we get the largest unit fraction less than the given unit fraction,w e can express it as a sum.

HOW TO FIND THE LARGEST UNIT FRACTION THAT IS LESS THAN THE GIVEN FRACTION?

Step 1: Find equivalent fraction for the given fraction such that you can manipulate the numerator into being a factor of the denominator.

23/35 = 46/70

Step 2: Split the 46 into 35+11

46/70 = 35/70 + 11/70

35/70 is nothing but ½ and that is the largest unit fraction less than 23/35

Therefore 23/35 = ½ + 11/70

= ½ + 10/70 + 1/70

= ½ + 1/7 + 1/70

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Anirved came up with all possible solutions for expressing unit fractions as sum of other unit fractions.

a. ½=⅓+⅙

b. ⅓=¼+1/12

⅓=⅕+1/15+1/15

c. ¼=⅕+1/20

¼=⅙+1/12

¼=1/7+1/14+1/28

h. 1/9=1/10+1/90

1/9=1/11+1/99+1/99

1/9=1/12+1/36

1/9=1/13+1/30+1/1,170

1/9=1/14+1/26+1/819

1/9=1/15+1/45+1/45

1/9=1/16+1/24+1/144

1/9=1/17+1/20+1/510+1/3,060

d. $\frac{1}{5}=\frac{1}{6}+1/30$

$\frac{1}{5}=1/7+1/35+1/35$

$\frac{1}{5}=\frac{1}{8}+1/20+1/40$

$\frac{1}{5}=1/9+1/12+1/180$

e. $\frac{1}{6}=1/7+1/42$

$\frac{1}{6}=\frac{1}{8}+1/24$

$\frac{1}{6}=1/9+1/18$

$\frac{1}{6}=1/10+1/15$

$\frac{1}{6}=1/11+1/22+1/33$

i. 1/10=1/11+1/110

1/10=1/12+1/60

1/10=1/13+1/65+1/130

1/10=1/14+1/35

1/10=1/15+1/30

1/10=1/16+1/40+1/80

1/10=1/16+1/32+1/160

1/10=1/17+1/34+1/85