Keep it Simple

Unit fractions (fractions which have numerators of 1) can be written as the sum of two <u>different</u> unit fractions.

For example, $\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

Charlie thought he'd spotted a rule and made up some more examples.

$\frac{1}{2} = \frac{1}{10} + \frac{1}{20}$ ✗ $\quad \frac{1}{3} = \frac{1}{4} + \frac{1}{12}$ ✓ $\quad \frac{1}{3} = \frac{1}{7} + \frac{1}{21}$ ✗ $\quad \frac{1}{4} = \frac{1}{5} + \frac{1}{20}$ ✓

Can you describe Charlie's rule? $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ $\quad a \times b = c$

Are all his examples correct? No

What do you notice about the sums that are correct? a & b are adjacent

Find some more correct examples.

How would you explain to Charlie how to generate lots of correct examples?

Alison started playing around with $\frac{1}{6}$ and was surprised to find that there wasn't just one way of doing this.

She found:

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$ $\quad \frac{1}{6} = \frac{1}{8} + \frac{1}{24}$ $\quad \frac{1}{6} = \frac{1}{9} + \frac{1}{18}$ $\quad \frac{1}{6} = \frac{1}{10} + \frac{1}{15}$

$\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$ (BUT she realised this one didn't count because they were not different.)

Charlie tried to do the same with $\frac{1}{8}$. Can you finish Charlie's calculations to see which ones work?

$\frac{1}{8} = \frac{1}{9} + \frac{?}{72}$ ✓ $\quad \frac{1}{8} = \frac{1}{10} + \frac{?}{40}$ ✓ $\quad \frac{1}{8} = \frac{1}{11} + \frac{?}{88}$ ✗

Can all unit fractions be made in more than one way like this?

Choose different unit fractions of your own to test out your theories.

6.2.25

Keep it Simple...

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

$\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$ PRIME

$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$

$\frac{1}{3} = \frac{1}{6} + \frac{1}{6}$ PRIME

$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$

$\frac{1}{4} = \frac{1}{6} + \frac{1}{12}$

$\frac{1}{4} = \frac{1}{8} + \frac{1}{8}$

$\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$

$\frac{1}{5} = \frac{1}{10} + \frac{1}{10}$ PRIME

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$

$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$

$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$

$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$

$\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$

$\frac{1}{7} = \frac{1}{8} + \frac{1}{56}$

$\frac{1}{7} = \frac{1}{14} + \frac{1}{14}$ PRIME

$\frac{1}{8} = \frac{1}{9} + \frac{1}{72}$

$\frac{1}{8} = \frac{1}{10} + \frac{1}{40}$

$\frac{1}{8} = \frac{1}{12} + \frac{1}{24}$ ~~$\frac{1}{8} = \frac{1}{16} + \frac{1}{16}$~~

$\frac{1}{9} = \frac{1}{10} + \frac{1}{90}$ $\frac{1}{9} = \frac{1}{12} + \frac{1}{36}$ ~~$\frac{1}{9} = \frac{1}{18} + \frac{1}{18}$~~

Not all unit fractions have this property. It doesn't work for prime denominators, like $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{11}$, etc.

I started with $\frac{1}{2}$. The next fraction is $\frac{1}{3}$, so $\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$.
Then I tried $\frac{1}{2} = \frac{1}{4} + \frac{1}{4}$ but the denominators were the same so it didn't count.
The reason why I stopped is that once I got to halfway if I continue it will always be a greater fraction.
For a fifth, for example, will use a non-unit fraction and for a sixth it will come up with a solution that we already have.

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Keep it Simple (table)

Number Property	÷ by	Fraction	Number of ways
Prime	2	$\frac{1}{2}$	1
Prime	3	$\frac{1}{3}$	1
Prime square	4	$\frac{1}{4}$	2
Prime	5	$\frac{1}{5}$	1
4 factors	6	$\frac{1}{6}$	4
Prime	7	$\frac{1}{7}$	1
Prime cube	8	$\frac{1}{8}$	3
Prime square	9	$\frac{1}{9}$	2
4 factors	10	$\frac{1}{10}$	4
Prime	11	$\frac{1}{11}$	1
6 factors	12	$\frac{1}{12}$	7
Prime	13	$\frac{1}{13}$	1
4 factors	14	$\frac{1}{14}$	4
4 factors	15	$\frac{1}{15}$	4
Prime^4	16	$\frac{1}{16}$	4
Prime	17	$\frac{1}{17}$	1
6 factors	18	$\frac{1}{18}$	7
Prime	19	$\frac{1}{19}$	1
6 factors	20	$\frac{1}{20}$	7
4 factors	21	$\frac{1}{21}$	4
4 factors	22	$\frac{1}{22}$	4
Prime	23	$\frac{1}{23}$	1

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Keep it Simple (table 2)

Number Property	÷ by	Fraction	Number of ways
8 factors (2 PRIME FACTS)	24	$\frac{1}{24}$	10
Prime square	25	$\frac{1}{25}$	2
4 factors	26	$\frac{1}{26}$	4
Prime cube	27	$\frac{1}{27}$	3
6 factors	28	$\frac{1}{28}$	7
Prime	29	$\frac{1}{29}$	1
8 factors (3 DIFF P FACTS)	30	$\frac{1}{30}$	12
Prime	31	$\frac{1}{31}$	1
Prime^5	32	$\frac{1}{32}$	5
4 factors	33	$\frac{1}{33}$	4
4 factors	34	$\frac{1}{34}$	4
4 factors	35	$\frac{1}{35}$	4

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Finding Pairs

n = denominator
p = prime factor
a = lowest prime factor
b = next prime factor

Number Property	Fractions	Pattern *	Number of ways
Prime	1/2, 1/3, 1/5, 1/7, 1/11, 1/13, 1/17, 1/19, 1/23, 1/29, 1/31 etc.	n+1	1
Prime Square	1/4, 1/9, 1/25, etc.	n+1, n+p	2
4 factors	1/6, 1/10, 1/14, 1/15, 1/21, 1/22, 1/26, 1/33, 1/34, 1/35, etc.	n+1, n+a, n+b, $n+a^2$	4
Prime Cube	1/8, 1/27, etc.	n+1, n+p, $n+p^2$	3
6 factors	1/12, 1/18, 1/20, 1/28, etc.	n+1, n+a, n+b, n+axb, $n+a^2$/$n+b^2$, 1 or 2 others	7
Prime^4	1/16, 1/81, etc.	n+1, n+p, $n+p^2$, $n+p^3$	4
8 factors	1/24, 1/30, etc.	n+1, n+a, $n+a^2$, $n+a^3$, n+b, n+axb, $n+a^2xb$, 3 others	10
		If 3 different prime factors, n+1, n+a, n+b, n+c, n+axb, n+axc, n+bxc, 5 others	If 3 different prime factors, 12

c = biggest prime factor

Number Property	Fractions	Pattern	Number of ways
Prime^5	$\frac{1}{32}, \frac{1}{243}$, etc.	$n+1$, $n+p$, $n+p^2$, $n+p^3$, $n+p^4$	5

* I found a pattern for the denominators that add to another unit fraction to make the original fraction

I worked out which fractions can be added to another unit fraction and wrote a tally to help me with the number of ways.

1/10	IIII							
	11	12	14	15				
	110	60	35	30				
1/12	~~IIII~~ II							
	13	14	15	16	18	20	21	
	156	84	60	48	36	30	28	
1/14	IIII							
	15	16	18	21				
	210	112	63	42				
1/15	IIII							
	16	18	20	24				
	240	90	60	40				
1/16	IIII							
	17	18	20	24				
	272	144	80	48				
1/18	~~IIII~~ II							
	19	20	21	22	24	27	30	
	342	180	126	99	72	54	45	
1/20	~~IIII~~ II							
	21	22	24	25	28	30	36	
	420	220	120	100	70	60	45	
1/24	~~IIII~~ ~~IIII~~							
	25	26	27	28	30	32	33	36
	600	312	216	168	120	96	88	72
	40	42						
	60	56						

1/30 ~~IIII~~ ~~IIII~~ II