

Pair of Numbers Solution for Nrich

Myself **Shubhangee (Facilitator)** had worked collaboratively on 'Pairs of Numbers' with a group of 13 students, in online mode, in 'Ganit Kreeda', Vicharvatika, India. The names of the students are:

Kimaya, Mrunmayee, Aarav, Ruhi, Amey, Siddharth, Reyansh, Vihaan, Vivaan, Rudra, Aaryavir, Rudravir and Swara.

All their responses are summarized here along with the pictures of their original solutions.

1. If you have ten counters numbered 1 to 10, how many can you put into pairs that add to 10?

Ans: 8

9+1, 8+2, 7+3, 6+4

Can you use them all? Why or why not?

No. We cannot use 5 and 10.

2. Now put the counters into pairs to make 12. Can you use them all? Why or why not?

No. We cannot use 1 and 6. As 10+2, 9+3, 8+4, 7+5 make 12.

3. Now put the counters into pairs to make 13. Can you use them all? Why or why not?

No. We cannot use 1 and 2. As 10+3, 9+4, 8+5, 7+6 make 13.

4. Now put the counters into pairs to make 11.

Can you use them all?

Why or why not?

Yes. We can make 10+1, 9+2, 8+3, 7+4 and 6+5.

Kids said that if we can choose a number (total), which is sum of the smallest and the biggest number then it is possible to use all of them.

Then they tested this by changing total numbers....

e.g for 1,2,3,4,5,6,7,8,9

If we choose total = 10...we cannot use 5, the middle number. Though both ending numbers are used.

Then they realised that it is possible to use all the numbers if there are even numbers of consecutive numbers and the total is the sum of the smallest and the biggest number.

For ex:

For 1 to 6, total = 7

For 1 to 100, total = 101 (1+100, 2+99....50+51)

For 1 to 50, total = 51 (1+50, 2+49....25+26)

Aarav's work:

Pairs of numbers h/w

A1 (i) We can put all the counters except 5 and 10.
(ii) We cannot put 5 and 10 because there is no pair for them.

A2 (i) We can put all the counters except 6 and 1.
(ii) We cannot put 6 and 1 because there is no pair for them.

A3 (i) We can put all the counters except 1 and 2.
(ii) We cannot put 1 and 2 because there is no pair for them.

A4 (i) We can put all the counters.
(ii) We

Here is Mrunmayee's work:

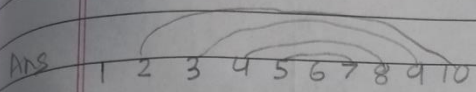
4) Now put the counters into pairs to make 11. Can you use them all? why or why not.

Ans

1 2 3 4 5 6 7 8 9 10

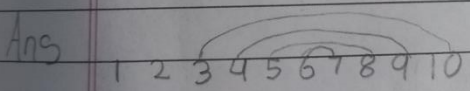
This will work because the middle, biggest and smallest numbers are used.

2) Now put the counters into pairs to make 12. Can you use them all? why or why not?



This will not work because there is no 11 and another 6 for 1 and 6

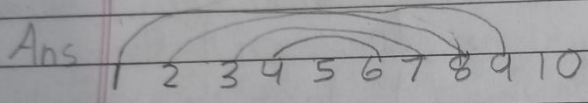
3) Now put the counters into pairs to make 13. Can you use them all? why or why not?



This will not work there is no 11 and 12 for 1 and 2

4) If you have ten counters numbered 1 to 10, how many can you put into pairs that add to 10?

Can you use them all? why or why not?



We cannot use all of them because 10 and 5 is left out because 5 and 10 don't have a pair

Observation:

If we multiply the middle number by 2 we will get the number we are suppose to make. This only works for even numbers.

Here is Vihaan's work:

① Pairs of numbers HW

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You can make 4 pairs.

1 No you can't use all the No.s because there are No.s from 1 to 10. To get the sum of 10 from 1 to 10 is impossible. The only way possible is $10 + 0.5$ also does not have a pair because there is No. 5 again. You need 2 5s to make 10.

2 No you can't use all the No.s because 1 does not have a pair and 1 can never have a pair in this case because $1 + 10 = 11$, we are adding the biggest No. possible, but still it doesn't work.

3 No you can't use all the No.s because 1 and 2 both don't have pairs. 1 needs 12 to a pair and 2 needs 11 to make a pair, but we only have No.s from 1 - 10 and 11 and 12 are not there.

②

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4 Yes this time you can use all the No.s because as the 1st and last No. make 11 then all the No.s will make without any No. getting no pair, because we go 1 down from 10 that is -1 and we go 1 No. higher than 1 that is +1, we all know +1 and -1 will give you 0, it will keep happening like +3, -3, 4, 4- and soon.

SELF I am going to share all my tricks. Now I am going to tell you how to find out if all No.s can be used or not.

1st conditions for all No.s being used that the sum has to be odd every time

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2nd condition the No. sum has to be a specific No. that is you have to + 1 to the biggest No. from the No.s.

3rd condition the No.s should all ways end in an even No.

Now I am going give reason.

Reason

1st condition = because 1 is an odd No. Odd + even = odd, you might think Odd + odd = Even will also work but no, cause if we are ending with an odd No, the middle No. is left.

right because even can be divided by 2.

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2nd condition = because as I had told if 1st and last No. make the sum then all No.s will be used as +1, -1, 2+, 2-, 3+, 3- and so on.

3rd condition = because to use all no.s if 1st and last No. make the sum +1, -1 + 2, -2 so on and odd + even = odd.

right
No.s that we don't use. I will give reason also.

1 If 1st and last don't make the sum try the No. after if it works then 1st No. is left but if it still doesn't work then 1st and the No. after that is left and so on